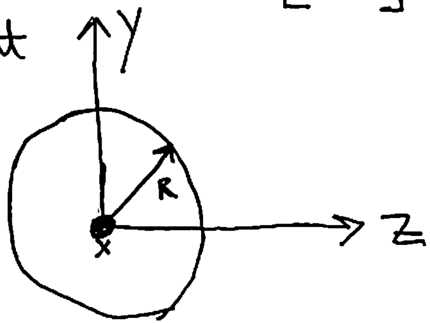
P2212 Jan 26 Example

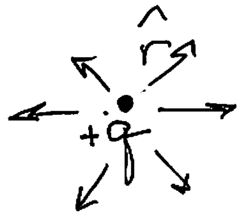
σ = area charge density , $[\sigma] = C/m^2$

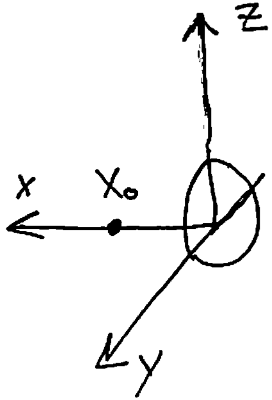
$\sigma = \sigma_0 r'^2$, $[\sigma_0] = \frac{[\sigma]}{[r^2]} = \frac{C/m^2}{m^2} = \frac{C}{m^4}$

$\sigma_0 > 0$

different r from $\hat{r}$ & r^2 in Coulomb's law



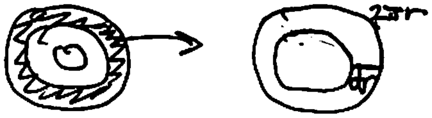




$\vec{E} = \frac{Kq}{r^2}\hat{r}$

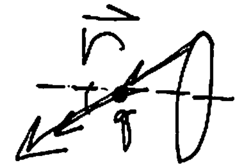
~~from source point pointing~~
pointing from source point towards field point

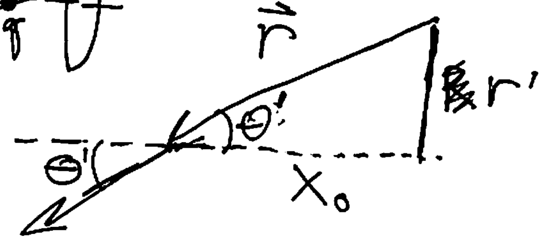
$d\vec{E} = \frac{K\,dq}{r^2}\hat{r}$, $dq = \sigma(r')\,dA$



From symmetry of λ about θ, one can conclude that $E_y = E_z = 0$. So compute E_x, ~~no θ dependence~~ which has no theta dependence.

$$dA = 2\pi r' dr'$$





$\cancel{\hat{r} = \frac{\vec{r}}{r}}$ $\cancel{\vec{r} =}$

just compute dE_x

$$dE_x = \frac{K dq}{r^2}\cos\theta', \quad \theta' = \cancel{\cos^{-1}\left(\frac{x_0}{\sqrt{x_0^2}}\right)} = \cos^{-1}\left(\frac{x_0}{\sqrt{x_0^2 + r'^2}}\right)$$

$= \cancel{\tan}$

$$= \frac{K dq}{r^2}\frac{x_0}{\sqrt{x_0^2 + r'^2}}, \quad r^2 = r'^2 + x_0^2$$

$$E_x = \int_{\substack{\text{over}\\\text{disk}\\\text{surface}}} dE_x = \int_0^R \frac{K\sigma_0 r'^2\, 2\pi r' dr'\, x_0}{(r'^2 + x_0^2)\sqrt{x_0^2 + r'^2}}$$

$$= K\sigma_0 (2\pi)\, x_0 \int_0^R dr' \frac{r'^3}{(r'^2 + x_0^2)^{3/2}}$$

$$\cancel{= 2\pi K\sigma_0 x_0 \frac{r' x_0}{\sqrt{r'^2 + x_0^2}}\Bigg|_0^R = 2\pi K\sigma_0 x_0^2 \frac{R}{\sqrt{R^2 + x_0^2}}}$$

$$= K\sigma_0 (2\pi)\, x_0 \frac{r'^2 + 2x_0^2}{\sqrt{r'^2 + x_0^2}}\Bigg|_0^R$$

$$= K\sigma_0 (2\pi)\, x_0 \left(\frac{R^2 + 2x_0^2}{\sqrt{R^2 + x_0^2}} - \frac{2x_0^2}{\cancel{x_0}}\right)$$

$$= 2\pi K\sigma_0 x_0 \left(\frac{R^2 + 2x_0^2}{\sqrt{R^2 + x_0^2}} - 2x_0\right)$$